Determination and improvement of bevel gear efficiency by means of loaded TCA

Dr. **J. Thomas**, Dr. **C. Wirth**, ZG GmbH, Germany

Abstract

Bevel and hypoid gears are widely used in automotive and industrial transmissions. Especially for the design of axle drives for passenger cars, trucks and busses gear efficiency becomes more and more important.

Today's most accurate calculation methods for tooth contact analysis (TCA) are based on numerical methods. An approach developed by ZG GmbH allows for the calculation of local efficiency grades, based on values determined by loaded TCA. Due to the local approach the micro geometry of the gear (based on machine settings) can be optimized regarding gear efficiency considering different speeds, loads and deflections in shafts.

This paper shows efficiency calculation for a practical Hypoid gear set of a typical passenger car axle drive. The effects of load cycle calculations and corresponding axis positions under load are taken into account as well as the possibilities of optimization of gear efficiency using the new calculation method. Possible conflicts between load capacity, noise (NVH) and gear efficiency are shown as well.

1. Basic relationships of forces in the gear mesh

Even though there was a report on the theory behind the determination of local efficiency already in [1], for proper understanding some of the basics are repeated here.

As soon as two mating flanks of a bevel or hypoid gear are in mesh, power is transferred by the forces acting on the flanks. Due to occurring friction in any considered contact point any sliding component causes friction force F_r. Frictional force F_r is always opposed to the sliding direction.

The forces are examined on an involute cylindrical gear shown in Figure 1. Two different mesh positions are considered regarding acting forces on the corresponding gear. The pinion is the driving member. At contact point #1 the normal force F_{n1} acts against the pinion flank. Sliding velocity at this point causes a friction force F_{r1} orientated tangential to flank and points towards the centre of pinion. Of course corresponding forces F_{r2} and F_{n2} act in opposite direction to F_{r1} and F_{n1}.

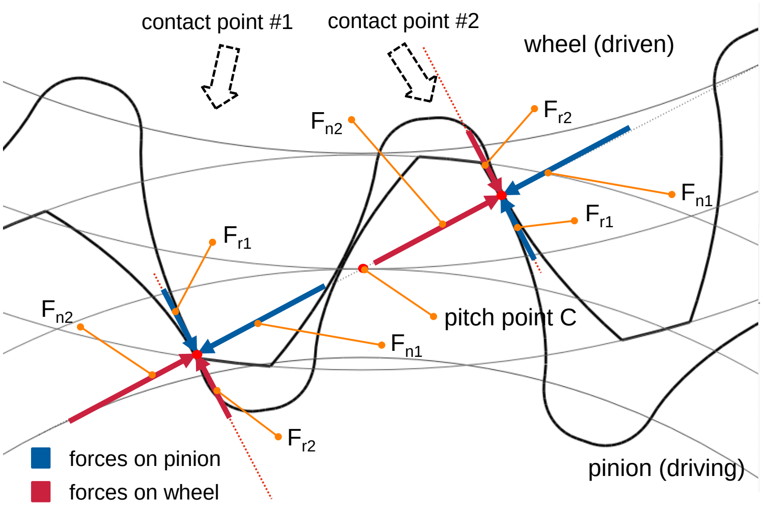

Figure 1: Force examination in two different contact points on a spur gear mesh

At contact point #2 normal forces occur in same direction as at contact point #1. However friction forces are opposed due to opposite sliding direction (direction is changing at pitch point C).

In Figure , the resulting torque components and corresponding lever arms are examined at contact point #1. Whereas lever arms for normal forces F_n around pinion and wheel axis are constant over the mesh, lever arms r_r for friction forces F_r vary.

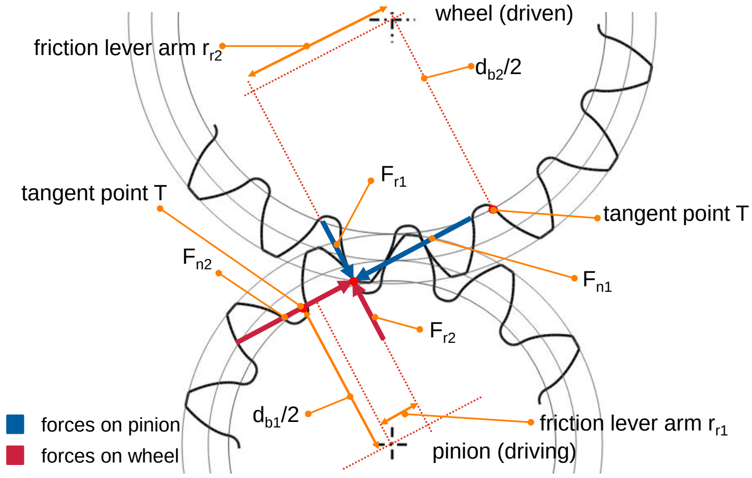

Figure 2: Force examination contact point #1

At contact point #2 lever arms will have a different length. For given normal forces F_n at any point in the contact area the resulting torque $T_{n1,2}$ around the pinion and wheel axes can be calculated according to Eq. 1. Resulting torque components $T_{r1,2}$ from the frictional forces have to be determined by Eq. 2 with corresponding lever arm $r_{r1,2}$. Due to torque equilibrium

(Eq. 3) the sum of T_n, T_r and external torque on the appropriate gear shaft has to be zero. For assumed constant normal forces the external torques on the gear shaft vary due to altering friction forces or, for constant torques on one shaft, normal forces vary. Therefore the ratio between the torque shafts is not constant as soon as friction is considered (see Eq. 4).

$$T_{n1,2} = F_{n1,2}\frac{d_{b1,2}}{2} \qquad \text{Eq. 1}$$

$$T_{r1,2} = -F_{r1,2}r_{1,2} \qquad \text{Eq. 2}$$

$$T_{1,2} + T_{n1,2} + T_{r1,2} = 0 \qquad \text{Eq. 3}$$

$$\frac{T_2}{T_1} = \frac{-T_{n2} - T_{r2}}{-T_{n1} - T_{r1}} = \frac{T_{n2} + T_{r2}}{T_{n1} + T_{r1}} \neq \frac{z_2}{z_1} = i \qquad \text{Eq. 4}$$

2. Local efficiency grade and resulting efficiency grade [1]

In general for calculating local efficiency grade η_k of any point Eq. 5 can be applied as soon as pinion is the driving member. For a driven pinion Eq. 6 has to be taken. Vectors $F_{n,k}$ and $F_{r,k}$ for the calculation of $T_{n,k}$ and $T_{r,k}$ have to be determined by a local TCA.

$$\eta_{k12} = -\frac{P_{ab,k}}{P_{an,k}} = \frac{T_{2,k}n_2}{T_{1,k}n_1} = \frac{T_{2,k}}{T_{1,k}i} = \frac{(T_{n2,k}+T_{r2,k})}{(T_{n1,k}+T_{r1,k})i} \qquad \text{Eq. 5}$$

$$\eta_{k21} = -\frac{P_{ab,k}}{P_{an,k}} = \frac{T_{1,k}n_1}{T_{2,k}n_2} = \frac{T_{1,k}}{T_{2,k}i} = \frac{(T_{n1,k}+T_{r1,k})i}{(T_{n2,k}+T_{r2,k})} \qquad \text{Eq. 6}$$

where

η_{k12}	[-]	is the efficiency grade for single contact point when the pinion is driving
η_{k21}	[-]	is the efficiency grade for single contact point when the wheel is driving
$P_{ab,k}$	[W]	is the output power of one single contact point
$P_{an,k}$	[W]	is the input power of one single contact point
$T_{1,2k}$	[Nm]	is the torque resulting from one single contact point
$T_{n1,2k}$	[Nm]	is the torque resulting from the normal force of one single contact point
$T_{r1,2k}$	[Nm]	is the torque resulting from the friction force of one single contact point
n_1	[1/min]	is the pinion RPM
n_2	[1/min]	is the wheel RPM
i	[-]	is gear ratio

The local efficiency grade of single points might be interesting for identifying flank areas causing low efficiency. For an approach of the resulting efficiency grade, all grades of all contact lines in mesh have to be determined in a first step. As Eq. 7 shows, all discrete calculated torques at the considered points have to be summed up for each contact line in mesh. A local efficiency grade for one contact line η_l can be introduced (Eq. 8).

$$T_{n1,l} = \sum_{n=1}^{K(l)} T_{n1,k} \qquad T_{r1,l} = \sum_{n=1}^{K(l)} T_{r1,k}$$
$$T_{n2,l} = \sum_{n=1}^{K(l)} T_{n2,k} \qquad T_{r2,l} = \sum_{n=1}^{K(l)} T_{r2,k} \qquad \text{Eq. 7}$$

$$\eta_l = -\frac{P_{ab,l}}{P_{an,l}} = \frac{T_{2,l}n_2}{T_{1,l}n_1} = \frac{T_{2,l}}{T_{1,l}i} = \frac{(T_{n2,l}+T_{r2,l})}{(T_{n1,l}+T_{r1,l})i} \qquad \text{Eq. 8}$$

where

η_l	[-]	is the efficiency grade of one single contact line
K	[-]	is the number of contact points along considered contact line
$P_{ab,l}$	[W]	is the output power of one single contact point
$P_{an,l}$	[W]	is the input power of one single contact point
$T_{1,2l}$	[Nm]	is the pinion / wheel torque
$T_{n1,2l}$	[Nm]	is the pinion / wheel torque from normal forces on considered contact line
$T_{r1,2l}$	[Nm]	is the pinion / wheel torque from friction forces on considered contact line

The resulting efficiency grade η_{res} can be achieved using Eq. 9.

$$\eta_{res} = 1 - \sum_{n=1}^{N}((1-\eta_l)Y_l) \qquad \text{Eq. 9}$$

where

Y_l	[-]	is the percentage of load that is transferred by the considered contact line

3. Calculation of local forces and torques [1]

The FVA- BECAL software (Bevel Gear Calculation) is used for the calculation of contact stress and tooth root stress distribution [1]. Using machine settings for any gear set, BECAL generates the geometry of the flank surface and tooth root. Analysing a virtual mesh loaded tooth contact analysis leads to the tooth root stresses and local Hertzian contact stresses. Deflections and deformations of housing, bearings and shafts can be considered as well. Wirth [5] developed “LokAna” (<u>loc</u>al <u>ana</u>lysis), a software program using results of BECAL as the input to analyze geometric conditions, load capacity (pitting, bending, subsurface fatigue), and other parameters. This program is used by ZG in designing and recalculating bevel and hypoid gear sets.

Thus local sliding velocities and local normal forces can be calculated and as soon as local friction coefficients are known (see following chapter), local friction forces as well. Finally "LokAna" calculates for local efficiency and resulting (mean) efficiency according to the above mentioned rules.

4. Calculation of friction coefficient [1]

The friction coefficient is calculated according to Klein [4]. Eq. 10 shows the formula and its influence parameters.

$$\mu_i = 0{,}036 \cdot \left(\sigma_{H,i}\right)^{0,10} \cdot \left(v_{\Sigma vert,i}\right)^{-0,20} \cdot \left(s_{x,i}\right)^{0,05} \cdot \left(C_\lambda \cdot \lambda_{z,i}\right)^{-0,10} \cdot C_L \cdot C_{RS} \qquad \text{Eq. 10}$$

where

μ_i	[-]	is the local friction coefficient
$\sigma_{H,i}$	[-]	is the local Hertzian contact stress
$V_{\Sigma,vert,i}$	[m/s]	is the local sum velocity perpendicular to contact line
$s_{an,I}$	[-]	is the local slip-roll ratio
C_λ	[-]	is the lubrication thickness factor
$\lambda_{z,i}$	[-]	is the roughness related film thickness
C_L	[-]	is the lubrication factor
C_{RS}	[-]	is the structure roughness factor

Unlike friction coefficients calculated according to [3] to determine a mean friction coefficient for the mesh, Klein developed a local applicable friction coefficient for bevel and hypoid gears. It is used in his scuffing load capacity calculation based on local TCA as well. The influences of important local parameters, such as the Hertzian stress, the sum velocity perpendicular to the contact line, the slip-roll ratio, and the roughness related film thickness are considered. The Hertzian stresses again are determined by BECAL. The sum velocities, the slip-roll ratio and the roughness related film thickness influencing hydrodynamic parameters in the contact are calculated by LokAna. An important influence is the orientation of the grinding grooves of both flanks to each other in the contact point.

5. Sample calculations

For sample calculations Hypoid gears were used with gear ratio u = 38/13, outer pitch diameter of the wheel d_{e2} = 190 mm, Hypoid offset a = 10 mm, wheel face width b_2 = 29 mm, normal design pressure angle α_n = 20°, and averaged mean spiral angle $(\beta_{m1} + \beta_{m2}) / 2$ = 35°.

Both a Face Milled (FM) and a Face Hobbed (FH) gear sets are considered. All calculations were done for pinion input torque of T1 = 350 Nm and pinion speed n1 = 1000 rpm. As lubricant a 75W-90 Oil with a temperature of 90°C was assumed. Tooth flank surface roughness Rz is 6 µm for all gears, neglecting special effects of grinding or lapping. Figure 3 shows results for calculated contact pattern, local efficiency grade and resulting mean efficiency of the FM gear.

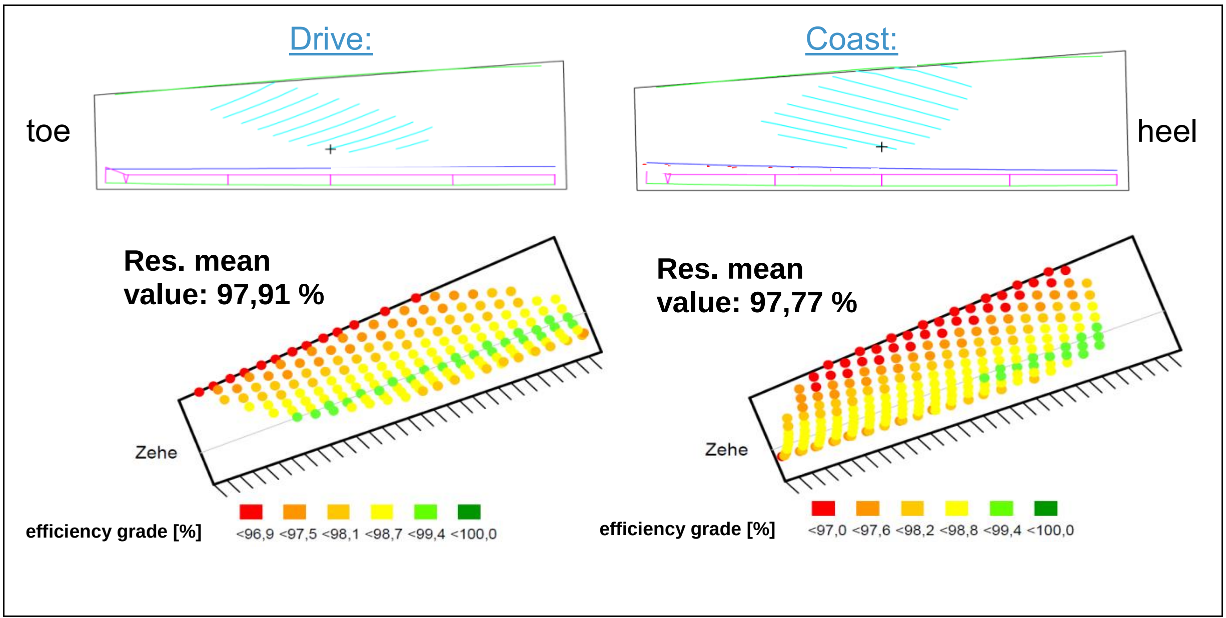

Figure 3: Contact pattern position, local and resulting mean efficiency of FM Hypoid gear

6. Relationship of efficiency and Hypoid offset

To evaluate the relationship between efficiency and Hypoid offset, 7 additional gear sets, FM and FH, were designed, for offsets a = 0, 1, 3, 5, 15, 20, and 25 mm. To get the pure influence of offset, all these gears show more or less same ease-off and contact pattern on drive and coast sides. The averaged mean spiral angle for all gear sets is set to $(\beta_{m1} + \beta_{m2}) / 2 = 35°$. Again calculations were done for same load, oil, and surface data.

Figure 4 shows the calculated resulting efficiency as a function of offset. It is clearly shown, that there are no fundamental differences between FM and FH gears, but the drive side always show up with higher efficiency then the coast side. This effect is well known and already was documented by Wech [6] in 1987, but typically could not be evaluated by established calculation methods. Higher offset values lead to higher losses as expected due to the fact that Hypoid gears always show sliding speed along face width, whereas local sliding velocities are zero on the pitch cone for bevel gears without offset. Figure 5 therefore shows local efficiency grades of 100% along the pitch cone for the bevel gear, whereas for the Hypoid gear with an offset a = 25 mm the local efficiency grades do not exceed 97,4%.

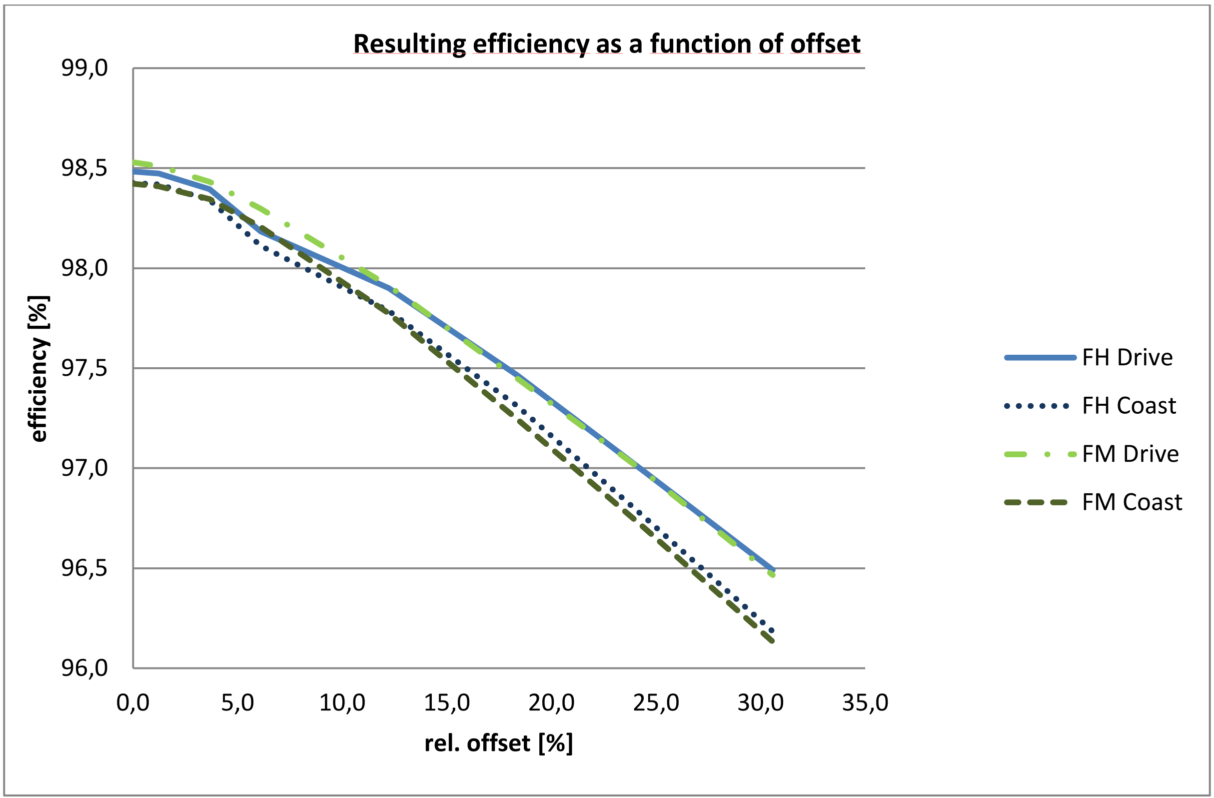

Figure 4: Relationship of resulting efficiency and Hypoid offset for example gear sets with equal size and Ease-Off

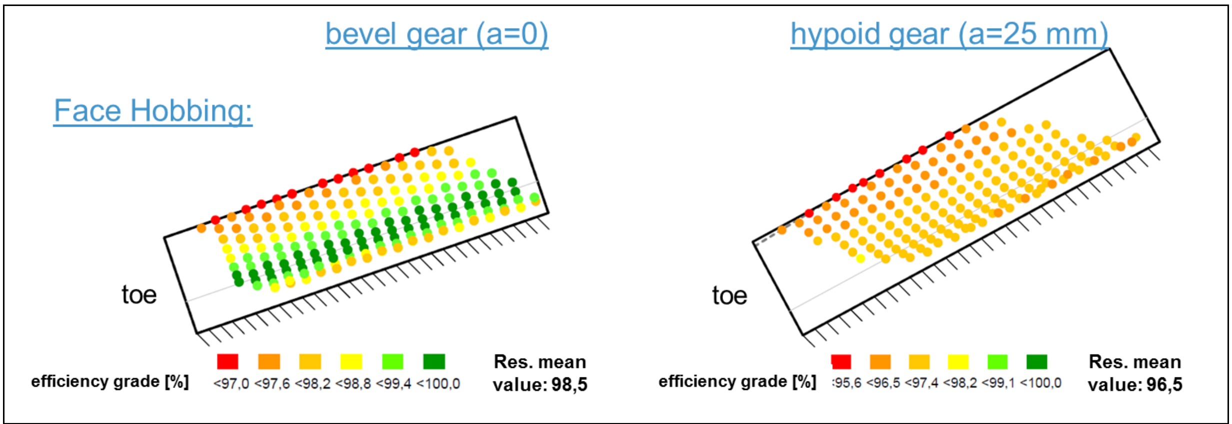

Figure 5: Local efficiency grades for FH bevel gear and FH Hypoid gear (a = 25 mm)

7. Relationship of efficiency and load

Figure 6 shows the resulting mean efficiency as a function of pinion torque. Efficiency decreases significantly with increasing load in bevel and Hypoid gears. This decrease is much more than it may be found on spur and helical gears and is caused by the influence of profile and lengthwise crowning usually found in bevel and Hypoid gears. Thus the loaded contact pattern grows with increasing load and more sliding points get in contact.

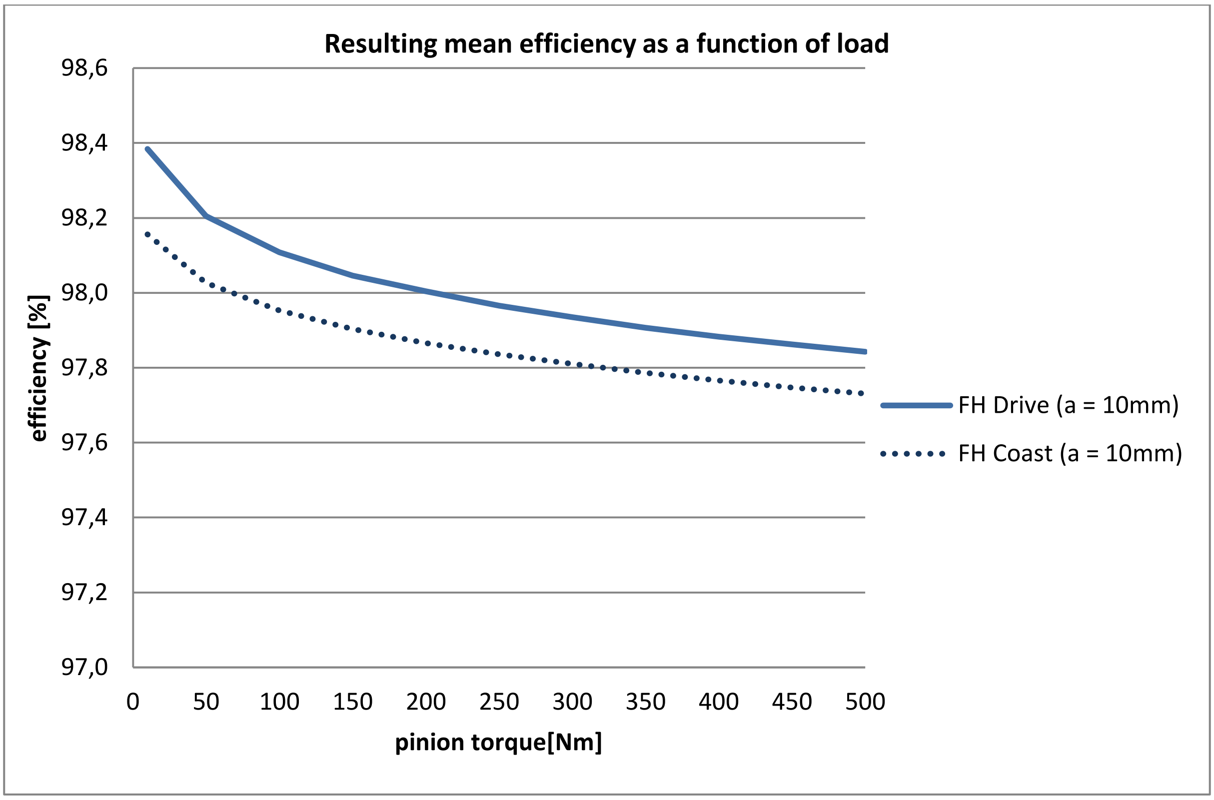

Figure 6: Relationship of resulting efficiency and load (pinion torque)

8. Improvement of gear efficiency by means of loaded TCA

The results of the new method take into account all effects of micro geometry of bevel and Hypoid gears on gear efficiency as efficiency is calculated locally point by point. Therefore all effects of changing contact pattern by design or by misalignments caused by load deformations of shafts, bearings, and housing are evaluated. Therefore this method now allows for optimizing the contact pattern for every single load case.

Left side of Figure 7 shows example of Figure 3 again. Graphs on right side of Figure 7 show new contact pattern, shifted to zone of high local efficiency while increasing profile crowning. Mean effective efficiency improves for shifted contact pattern as expected.

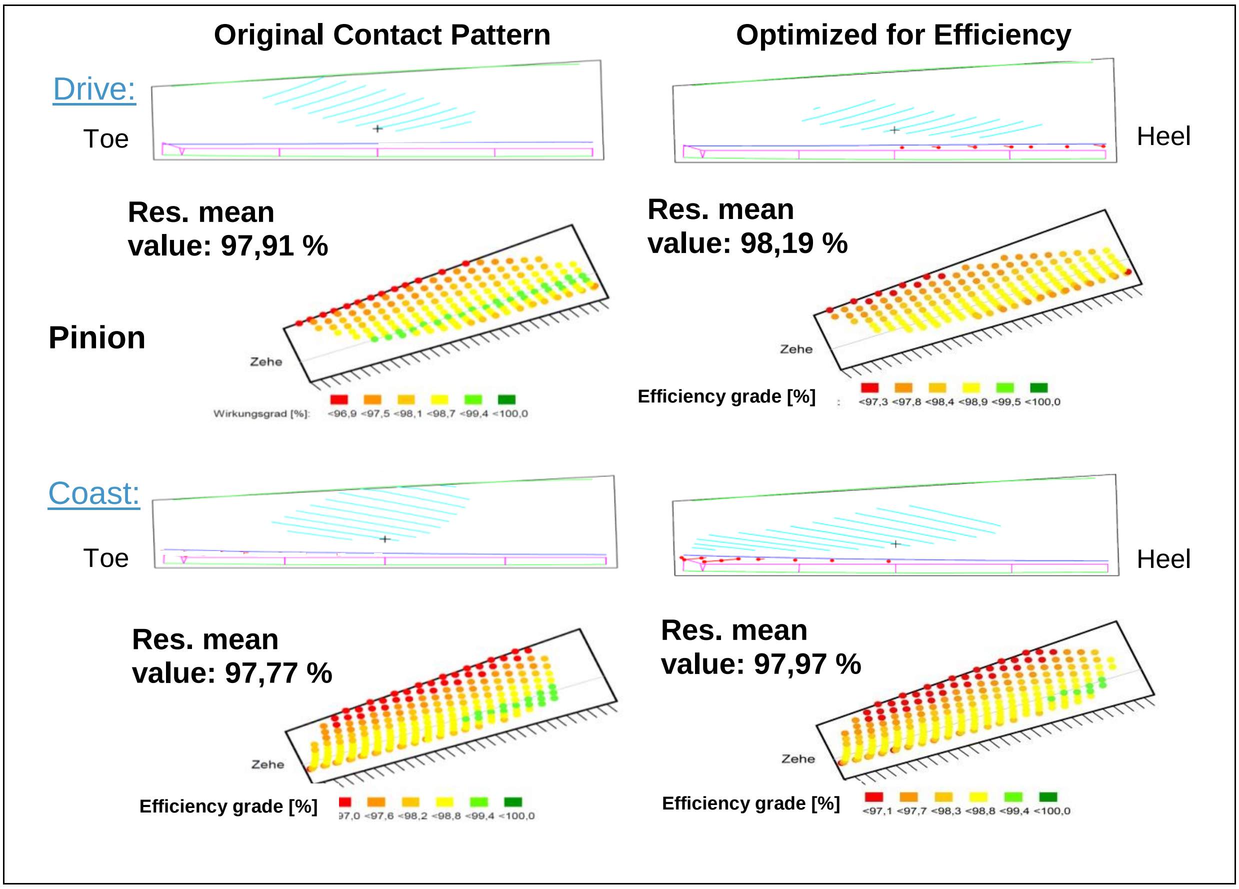

Figure 7: Original Contact Pattern and modified for higher efficiency of FM Hypoid gear

The FH example (see Figure 8) shows once more the strong influence of profile crowning. Here more or less profile crowning was increased only and thus main zone of contact pattern now is within zone of high local efficiency, as profile crowning is located along pitch cone. As nearby pitch cone the lowest sliding speeds will occur, highest local efficiency is located at same zone. Thus mean effective efficiency is improved by increasing profile crowning. Although this positive effect on efficiency, this measure will lead to some disadvantage in same time (see chapter 9).

9. Load cycle calculation and conflicting aims such as NVH and load capacity

All improvements of efficiency shown before are made for one specific load using related axis deviations only. As soon as there may be given load cycles, this optimisation has to be done for all loads and relating axis deviations of the complete load spectrum. It may happen, that optimisation of resulting mean efficiency for all load levels will be impossible. In such cases run-time of each load level has to be taken into account and the mean efficiency has to be weighted in similar way as this is done for load carrying capacity as well. Final result will be an “equivalent mean gear efficiency”.

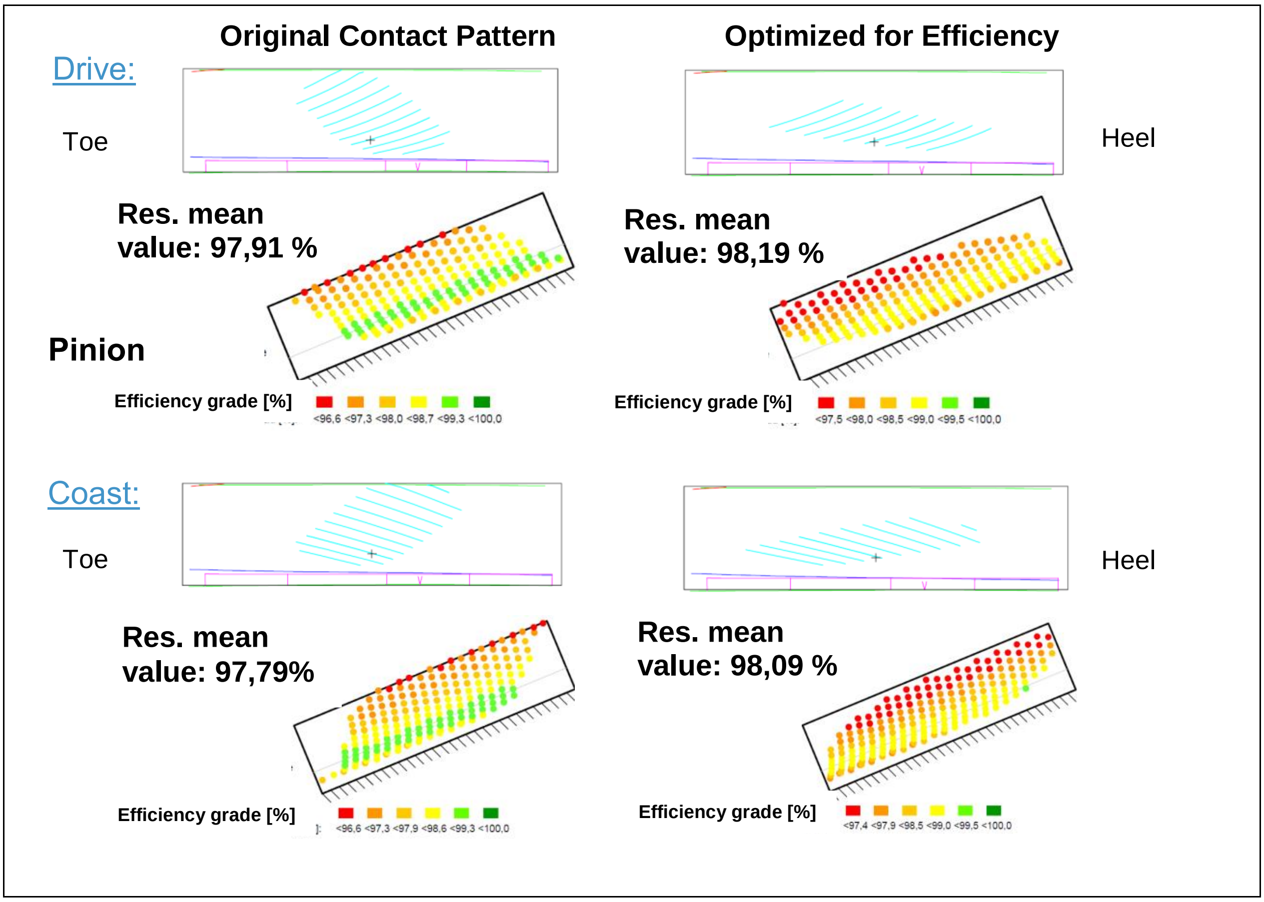

Figure 8: Original Contact Pattern and modified for higher efficiency of FH Hypoid gear

There may be other conflicting aims, such as noise behaviour (NVH) and load carrying capacity, which have to be taken into account as well while optimizing efficiency. Just increasing profile crowning usually will cause higher transmission error. This can be shown within Figure 9. The unloaded transmission error of the FM gear, optimized for efficiency, is nearby doubled compared to original version. But for NHV finally the loaded transmission error will be decisive and here is nearly no difference in amplitude. Therefore within this concrete example there will be no influence of higher profile crowning on NVH for the related example torque. Of course this may differ for other load stages.

It is important to check the influence of efficiency optimisation on stresses too. For the FM gear set the influence is shown within Figure 10. Whereas pressure stays within same range and is decreasing a little bit, tooth root stress is increasing within this example. It has to be decided by the designer, if this may be tolerable or not.

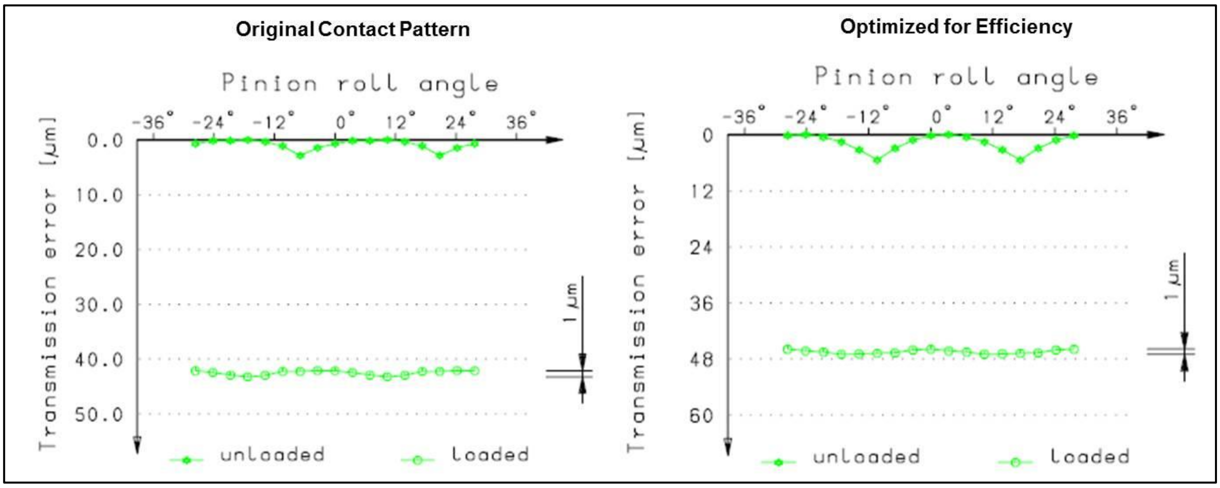

Figure 9: Comparison of transmission error for FM Hypoid gear example

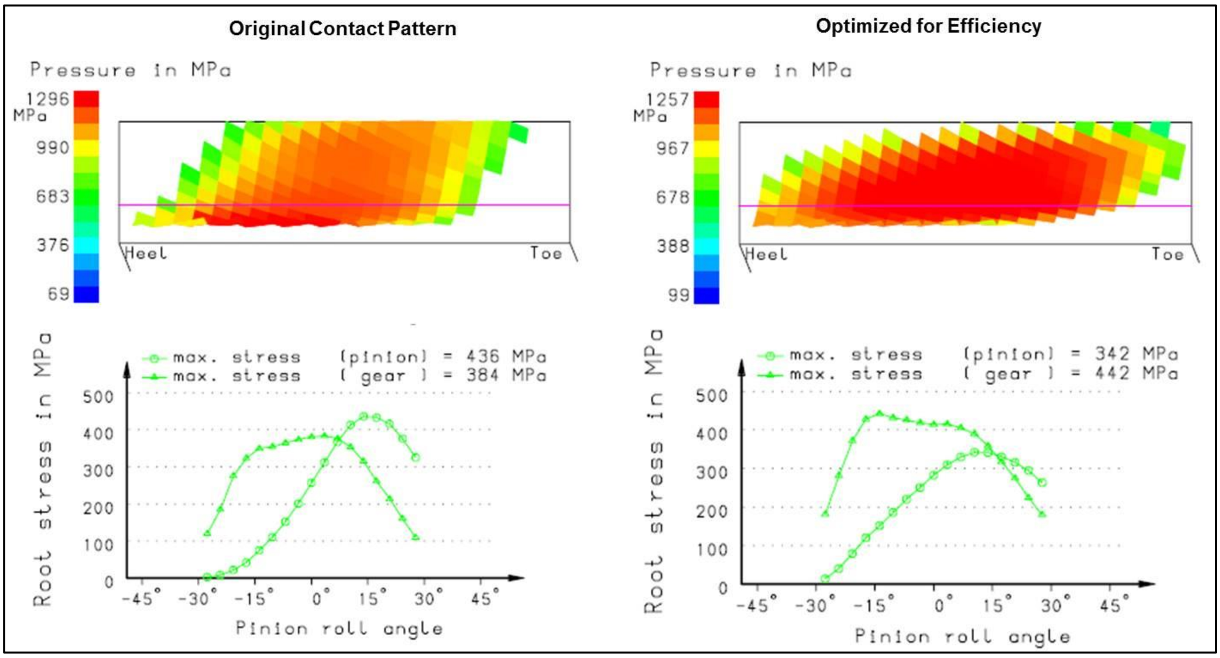

Figure 10: Comparison of pressure and tooth root stress for FM Hypoid gear example

10. Summary

Within this paper it could be demonstrated, that the approach for calculation of local efficiency grades developed by ZG GmbH allows for optimizing gear efficiency of bevel and Hypoid gears considering different speeds, loads and deflections in shafts. For load cycle calculations all load stages have to be considered. As there may be aims conflicting to gear efficiency, there always has to be found a compromise to get best contact pattern for all considered load cases, which will show best gear efficiency while fulfilling all needs regarding NVH and load capacity.

References

[1] Wirth, C.; Thomas, J.: Local efficiency grade of bevel and hypoid gears. VDI-Berichte Nr. 2255, 2015.

[2] Hutschenreiter, B., Schäfer, S.: BECAL – Programm zur Berechnung der Zahnflanken- und Zahnfußbeanspruchung an Kegel- und Hypoidgetrieben bei Berücksichtigung von Relativlage und Flankenmodifikationen (Version 4.1.0). FVA-Forschungsheft Nr. 1037. Forschungsvereinigung Antriebstechnik e.V.

[3] ISO/TR 13989: Calculation of scuffing load capacity of cylindrical, bevel and hypoid gears, part 1 and 2, 2000.

[4] Klein, M.: Zur Fresstragfähigkeit von Kegelrad- und Hypoidgetrieben. Dissertation TU München, 2012.

[5] Wirth, Ch.: Zur Tragfähigkeit von Kegelrad- und Hypoidgetrieben. Dissertation TU München, 2008.

[6] Wech, L.: Untersuchungen zum Wirkungsgrad von Kegelrad- und Hypoidgetrieben-Dissertation TU München, 1987.